# Polar Decomposition and Interference Model Applied to Metallic Nanospheres 

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#### Abstract

We have successfully applied the polar decomposition (PD) to the scattering matrix of coupling metallic nanospheres. The Discrete Dipole Approximation method (DDA) has been used as an intermediate tool to calculate these matrices. We also present a simple model based on the interference to justify the presence of a minimum in the scattered intensity. It is also shown how PD provides us with a frame in which the scattering systems can be characterized by independent parameters representing magnitudes of simple (virtual) elements constituting an alternative to conventional Mueller Matrix analysis methods.


## System Geometry and Numerical Method

The scattering system we analyze consists of two silver spheres ( $n=0,135+3,988 i$, $\lambda=633 \mathrm{~nm}$ ) of radius $r=0.1 \lambda$, and with gap distances ranging from $0.1 \lambda$ to $0.8 \lambda$. We have considered three different geometries (Fig. 1 a , b and c) all illuminated by a monochromatic plane wave of $\lambda=633 \mathrm{~nm}$. We numerically obtain the elements of the Mueller matrix by using the discrete dipole approximation (DDA) [1], which is a computational procedure suitable for studying scattering and absorption of EM radiation by particles with size of the order or less than the wavelength of the incident light [2].

The scattering matrices obtained from this method have been post-processed in all cases with an algorithm that performs the PD [3]. After testing the purity of the matrices [4], it was found that in the cases analyzed, the Mueller matrices obtained were pure. This means that the system does not produce any depolarization. Therefore, the system matrix can be decomposed as $M_{4 x 4}=M_{\Delta} M_{R} M_{D}$, where $M_{\Delta}$, the depolarization matrix, is the Identity ${ }_{4 x 4}$ in our case, $M_{R}$ is the retardance matrix and $M_{D}$ is the diattenuation matrix. Due to the symmetry properties, and as a result of the PD, we can decompose our problem in an equivalent system composed by an ideal diattenuator aligned with the scattering plane, $M_{D}(t)$, and with the fast axis of a lineal retarder, $M_{R}(\varphi)$.

If we examine in detail the polarimetric properties of our system and making use of the PD, we can describe the behavior of our system by just considering three independent parameters, the total system transmittance $\left(M_{I I}\right)$, the transmission along one of the diattenuator axes $(t)$ and the phase shift introduced by the retarder $(\varphi)$. Once the meaning of the PD parameters is well understood, this polarimetric method provides us with a more handy tool to approach the analysis of the system.

Following the interferencial analysis proposed in Ref. [5], we can carry out a simple model based on two spot scatterers (Fig. 1 d ), where we can evaluate the angular position of the minimum in $t$, corresponding to phase shifts and strong changes in the linear polarization degree ( $P_{L}=-M_{12} / \mathrm{M}_{11}$, and $P_{L}=1-2 t$ in our case). This model is summarized in equations (1.a) and (1.b), and is a result of the phase lags introduced by the optical path difference ( $\Lambda$ and $\Omega$ ):
(1.a) X-geometries: $(\Omega-\Lambda)=(2 \mathrm{n}+1) \pi=\left(\frac{2 \pi \mathrm{~d}}{\lambda}\right)(1-\cos (\theta))$
(1.b) Z-geometries:
$\Lambda=(2 \mathrm{n}+1) \pi=\left(\frac{2 \pi \mathrm{~d}}{\lambda}\right) \sin (\theta)$
where $I_{T}$ is the total scattered intensity in a $\theta$ direction, $n$ is the minima order and $d$ is the scatterers separation.

## Results and Conclusions

As shown in Fig.2, due to the strong interaction [5], small separations between spheres change the position of the minimum and soften the visibility in both X and Z geometries. However, when separation increases, interaction decreases, and therefore the model provides with more accurate values of minimum positions. Moreover, Y geometries do not introduce any dephase in the light spread along the scattering plane, and only shows a small deviation from the single sphere scatter when the distances between spheres are smaller.

Finally, just remark that PD magnitudes can fully describe the optical system behavior, and can be applied to other geometries, no matter its complexity. The sixteen elements of the Mueller Matrix can be reduced to a smaller number of independent ones, the same number that PD method requires. These parameters are easy to use and also represent magnitudes of simple virtual elements which improve the understanding of the processes involved in complex scattering systems.

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Figure 1


Figure 2


Some graphical results: Dotted Gray Line $\rightarrow$ Interference Model Predicted Minimum

