SUPERCONDUCTOR-FERROMAGNET NANOSTRUCTURES

A. Buzdin

Institut Universitaire de France, Paris

and Condensed Matter Theory Group, University of Bordeaux
OUTLINE

• Mechanisms of magnetism and superconductivity interaction.
• Electromagnetic interaction in S/F heterostructures.
• Proximity effect in superconductor-ferromagnet systems. Josephson Π-junction.
• Spin-valve effect. Domain wall superconductivity.
• Coupling between magnetic moment and Josephson current in $\varphi_0$-junctions.
• The way to superconducting spintronics. Possible applications.
Supraconductivity

Since the discovery by Heike Kamerlingh Onnes
6 Nobel Prizes.
Many important applications.

Magnetism

4 Nobel Prizes.

Recent Nobel Prize: A. Fert and P. Grunberg

Spintronics
Is it possible to couple magnetism and superconductivity?

Superconducting Cooper pairs
Antagonism of magnetism (ferromagnetism) and superconductivity

- Orbital effect (Lorentz force)

\[ p \] \[ F_L \] \[ -p \] \[ F_L \] \[ B \]

Electromagnetic mechanism (breakdown of Cooper pairs by magnetic field induced by magnetic moment)

- Paramagnetic effect (singlet pair)

\[ S_z = +1/2 \] \[ S_z = -1/2 \]

\[ \mu_B H \sim \Delta \sim T_c \]

\[ I \left( \vec{S} \cdot \vec{S} \right) \approx T_c \]

Exchange interaction
FERROMAGNETIC UNCONVENTIONAL (TRIPLET) SUPERCONDUCTORS

Triplet pairing

UGe$_2$ (Saxena et al., 2000) and URhGe (Aoki et al., 2001)

Very low T$_c$ < 1 K, clean limit…
The coexistence of singlet superconductivity and ferromagnetism is basically impossible in the same compound but may be easily achieved in artificially fabricated superconductor/ferromagnet heterostructures.

Due to the proximity effect, the Cooper pairs penetrate into the F layer and we have the unique possibility to study the properties of superconducting electrons under the influence of the huge exchange field.

The Josephson junctions with ferromagnetic layers reveal many unusual properties quite interesting for applications, in particular the so-called \( \pi \)-Josephson junction (with the \( \pi \)-phase difference in the ground state).
Superconductor-Ferromagnet hybrids

Interaction between superconductor (S) and ferromagnet (F):

1. Exchange interaction via the proximity effect (spin valve, $\pi$-junctions,...)

2. Electromagnetic interaction

Varying in the controllable manner the thicknesses of the ferromagnetic and superconducting layers it is possible to change the relative strength of two competing ordering. Interesting effects at the nanoscopic scale.
Domain wall superconductivity in hybrid S/F systems or ferromagnetic superconductors

S/F bilayer with domain structure (Buzdin, Melnikov, PRB, 2002)

Phase diagrams of S/F systems with periodic domain structures

This domain wall superconductivity has been observed on experiment by Moschalkov et al. (Nature Material, 2005).
Domain wall superconductivity

\[ H > 0 \]

\[ H = 0 \]

\[ H < 0 \]

Sample characterization

W. Gillijns et al.,
FIS in films with magnetic Co/Pd dots

Nanoengineered magnetic-field-induced superconductivity, PRL 90, 197006 (2003) + Focus
$\infty$ possible states

TUNABLE FIELD INDUCED SUPERCONDUCTIVITY
Werner Gillijns, Alejandro V. Silhanek, Victor V. Moshchalkov,

R(H) vs M
Superconducting order parameter behavior in ferromagnet

Standard Ginzburg-Landau functional:

\[ F = a|\Psi|^2 + \frac{1}{4m} |\nabla \Psi|^2 + \frac{b}{2} |\Psi|^4 \]

The minimum energy corresponds to \( \Psi = \text{const} \)

The coefficients of GL functional are functions of internal exchange field \( h \)!

Modified Ginzburg-Landau functional!:

\[ F = a|\Psi|^2 - \gamma |\nabla \Psi|^2 + \eta |\nabla^2 \Psi|^2 + \ldots \]

The non-uniform state \( \Psi \sim \exp(iqr) \) will correspond to minimum energy and higher transition temperature
\[ F = (a - \gamma q^2 + \eta q^4) \left| \Psi_q \right|^2 \]

\( \Psi \sim \exp(iqr) \) - Fulde-Ferrell-Larkin-Ovchinnikov state (1964).
Only in pure superconductors and in the very narrow region.
The total momentum of the Cooper pair is 
\(-(k_F - \delta k_F) + (k_F - \delta k_F) = 2 \delta k_F\)
Proximity effect in a ferromagnet?

In the usual case (normal metal):

\[ a\Psi - \frac{1}{4m} \nabla^2 \Psi = 0, \]  
and solution for \( T > T_c \) is \( \Psi \propto e^{-qx} \), where \( q = \sqrt{4ma} \)

In \textbf{ferromagnet} (in presence of exchange field) the equation for superconducting order parameter is different

\[ a\Psi + \gamma \nabla^2 \Psi - \eta \nabla^4 \Psi = 0 \]

Its solution corresponds to the order parameter which decays with \textbf{oscillations}! \( \Psi \sim \exp[-(q_1 \pm iq_2)x] \)

\textbf{Wave-vectors are complex!} \textbf{They are complex conjugate and we can have a real } \Psi. \]

Order parameter changes its \textbf{sign}!
Proximity effect as Andreev reflection

Classical Andreev reflection
(September, 2008, Miraflores, Spain)

Quantum Andreev reflection

$p_F^\uparrow \neq p_F^\downarrow$

$h \gg T_c$
Remarkable effects come from the possible shift of sign of the wave function in the ferromagnet, allowing the possibility of a «π-coupling» between the two superconductors (π-phase difference instead of the usual zero-phase difference)

\[ \xi_f = \sqrt{D_f / h} \propto (1-10) \text{nm} \]

h-exchange field, Df-diffusion constant
The oscillations of the critical temperature as a function of the thickness of the ferromagnetic layer in S/F multilayers has been predicted in 1990 and later observed on experiment by Jiang et al. PRL, 1995, in Nb/Gd multilayers
SF-bilayer $T_c$-oscillations

V. Zdravkov, A. Sidorenko et al
PRL (2007)

$\text{Nb-Cu}_{0.41}\text{Ni}_{0.59}$

Ryazanov et al. JETP Lett. 77, 39
(2003) $\text{Nb-Cu}_{0.43}\text{Ni}_{0.57}$

$\lambda_{\text{ex}}$ largest $T_c$-suppression

$d_{\text{Fmin}} = (1/4) \lambda_{\text{ex}}$
Josephson effect

\[ \Psi_1 = |\Psi_1| \exp(i\theta_1) \]
\[ \Psi_2 = |\Psi_2| \exp(i\theta_2) \]
\[ |\Psi_1| = |\Psi_2| \]

Superconducting phase difference:
\[ \varphi = \theta_1 - \theta_2 \]

Josephson relations
\[ I_s = I_c \sin \varphi \]
\[ V = \frac{\hbar}{2e} \frac{d\varphi}{dt} \]

Electromagnetic radiation at the frequency \( f \)
\[ f = \frac{V}{\Phi_0} \]
S-F-S Josephson junction in the clean/dirty limit

Damping oscillating dependence of the critical current $I_c$ as the function of the parameter $\alpha = \hbar d_F / v_F$ has been predicted.

(Buzdin, Bulaevskii and Panjukov, JETP Lett. 81)

$h$-exchange field in the ferromagnet, $d_F$ - its thickness

$J(\phi) = I_c \sin \phi$

$E(\phi) = - I_c (\Phi_0 / 2\pi c) \cos \phi$
Theory of S/F/S systems in dirty limit

Analysis on the basis of the Usadel equations

\[- \frac{D_f}{2} \nabla^2 F_f(x, \omega, h) + (\omega + i h) F_f(x, \omega, h) = 0\]

\[G_f^2(x, \omega, h) + F_f(x, \omega, h) F_f^*(x, \omega, -h) = 1\]

leads to the prediction of the oscillatory-like dependence of the critical current on the exchange field $h$ and/or thickness of ferromagnetic layer. (Buzdin and Kuprianov, 1991)
The oscillations of the critical current as a function of temperature (for different thickness of the ferromagnet) in S/F/S trilayers have been observed on experiment by Ryazanov et al. 2000, PRL

Later more S/F/S junctions has been observed, see for example a transition as a thickness of a ferromagnetic (NiPt) layer by Kontos et al. 2002, PRL
Phase-sensitive experiments

$\pi$-junction in one-contact interferometer

0-junction
minimum energy at 0

$\pi$-junction
minimum energy at $\pi$

$I = I_c \sin(\pi + \phi) = -I_c \sin \phi$

$E = E_J[1 - \cos(\pi + \phi)] = E_J[1 + \cos \phi]$

$2\pi L I_c > \Phi_0 / 2$

$\phi = \pi = (2\pi / \Phi_0) \int_{l} A dl$

$= 2\pi \Phi / \Phi_0$

Spontaneous circulating current
in a closed superconducting loop
when $\beta_L > 1$ with NO applied flux

$\beta_L = \Phi_0 / (4\pi L I_c)$

$\Phi = \Phi_0 / 2$

Bulaevsky, Kuzii, Sobyanin, JETP Lett. 1977
Current-phase experiment.
Two-cell interferometer
Cluster Designs (Ryazanov et al.)

2 x 2
unfrustrated
fully-frustrated
checkerboard-frustrated

6 x 6
fully-frustrated
checkerboard-frustrated

30 μm
2 x 2 arrays: spontaneous vortices

Fully frustrated

Checkerboard frustrated
Scanning SQUID Microscope images
(Ryazanov et al.)

\[ I_c \]

\[ T_{\pi} \]

\[ T = 1.7K \]
\[ T = 2.75K \]
\[ T = 4.2K \]
Critical current density vs. F-layer thickness (V.A.Oboznov et al., PRL, 2006)

\[ I_c = I_{c0} \exp \left(-\frac{d_F}{\xi_{F1}}\right) \left| \cos \left(\frac{d_F}{\xi_{F2}}\right) + \sin \left(\frac{d_F}{\xi_{F2}}\right) \right| \]

Spin-flip scattering decreases the decaying length and increases the oscillation period.

\[ \xi_{F2} > \xi_{F1} \]

\( d_F >> \xi_{F1} \)

Nb-Cu\textsubscript{0.47}Ni\textsubscript{0.53}-Nb

"0"-state

I = I_c \sin \phi

I = I_c \sin(\phi + \pi) = -I_c \sin(\phi)
Critical current vs. temperature
(0-π- and π-0- transitions)

Nb-Cu$_{0.47}$Ni$_{0.53}$-Nb

d$_{F1}$=10-11 nm
d$_{F2}$=22 nm

(V.A.Oboznov et al., PRL, 2006)
FIG. 1: Geometry of $S/F^\prime/F/F'''/S$ junction. The arrows indicate non-collinear orientations of magnetizations in each layer with thickness $d_L$, $d$, $d_R$, respectively ($L = d_L + d + d_R$).

$$\xi_f \ll L \ll \xi_0$$

$$e R_F I_c = -\frac{2\Delta(T)^2 \hbar^2}{\pi^3 T_c^3} \sin \theta_R \sin \theta_L$$

(+ small term)

FIG. 2: Critical current induced by long range triplet proximity effect in S/F'/F/F''/S junction, in units of $\frac{\pi G \Delta(T)^2}{4eT_c}$, for varying length of F' and F'' layers, at $d_L = d_R \sim \xi_f \ll d \ll \xi_0$, and for different orientations of the magnetization in the layers.

Rather sharp maximum of the critical current at $d_L = d_R = \xi_f$
Firstly the F/S/F trilayers has been studied experimentally in 1968 by Deutscher et Meunier.
In this special case, we see that the critical temperature of the superconducting layers is reduced when the ferromagnets are polarized in the same direction.
In the dirty limit, we used the quasiclassical Usadel equations to find the new critical temperature $T^*_c$.
We solved it self-consistently supposing that the order parameter can be taken as:

$$\Delta = \Delta_0 \left(1 - \frac{x^2}{L^2}\right)$$

with $L \gg d_s$

Buzdin, Vedyaev, Ryazhanova, Europhys Lett. 2000,

In the case of a perfect transparency of both interfaces

$$d^* = \gamma \sqrt{\frac{h}{D_n}} \frac{D_s}{4\pi T_c}$$

$$\ln \left(\frac{T_c^{\uparrow\downarrow}}{T_c}\right) = \Psi \left(\frac{1}{2}\right) - \text{Re} \Psi \left(\frac{1}{2} + \frac{d^* T_c}{d_s T_c^{\uparrow\downarrow}} (1+i)\right)$$

$$\ln \left(\frac{T_c^{\uparrow\uparrow}}{T_c}\right) = \Psi \left(\frac{1}{2}\right) - \Psi \left(\frac{1}{2} + \frac{d^* T_c}{d_s T_c^{\uparrow\downarrow}}\right)$$
Recent experimental verifications

<table>
<thead>
<tr>
<th>AF</th>
<th>Fe_{50}Mn_{50} (8 nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>Ni (7 nm)</td>
</tr>
<tr>
<td>S</td>
<td>Nb (d_s)</td>
</tr>
<tr>
<td>F2</td>
<td>Ni (7 nm)</td>
</tr>
</tbody>
</table>

« free » F layer

F layer with fixed magnetization

P state

AP state

CuNi/Nb/CuNi
Gu, You, Jiang, Pearson, Bazaliy, Bader, 2002

Ni/Nb/Ni
Moraru, Pratt Jr, Birge, 2006

\[ T \text{(K)} \]
Evolution of the difference between the critical temperatures as a function of interfaces’ transparency

\[ \gamma_B = 0 \]
Infinite transparency

\[ \gamma_B \approx 5 \]
Finite transparency

\[
\begin{align*}
\ln \left( \frac{T_{c \uparrow \uparrow}}{T_c} \right) &= \Psi \left( \frac{1}{2} \right) - \text{Re} \Psi \left( \frac{1}{2} + \frac{\tilde{d} T_{c \uparrow \uparrow}^*}{d_s T_{c \uparrow \uparrow}^*} (1+i) \right) \\
\ln \left( \frac{T_{c \uparrow \downarrow}}{T_c} \right) &= \Psi \left( \frac{1}{2} \right) - \Psi \left( \frac{1}{2} + \frac{\tilde{d} T_{c \uparrow \downarrow}^*}{d_s T_{c \uparrow \downarrow}^*} \right)
\end{align*}
\]

\[
\tilde{d} = \frac{D_s}{4\pi T_c} \frac{\gamma \sqrt{\frac{h}{D_n}}}{1 + (1+i)^{\gamma_B}} \sqrt{\frac{h}{D_n}}
\]
Similar physics in F/S bilayers

In practice, magnetic domains appear quite easily in ferromagnets

\[ \text{Ni}_{0.80}\text{Fe}_{0.20}/\text{Nb} \ (20\text{nm}) \]

Thin films: Néel domains

Rusanov et al., PRL, 2004

"w": width of the domain wall

Localized (domain wall) superconducting phase.

Different mechanism for the $\varphi_0$ - Josephson junction realization.

Recently the broken inversion symmetry (BIS) superconductors (like CePt$_3$Si) have attracted a lot of interest.

Very special situation is possible when the weak link in Josephson junction is a non-centrosymmetric magnetic metal with broken inversion symmetry!

Suitable candidates: MnSi, FeGe.

Josephson junctions with time reversal symmetry: $j(-\varphi) = -j(\varphi)$;
i.e. higher harmonics can be observed $\sim j_n \sin(n\varphi)$ – the case the $\pi$ junctions.

Without this restriction a more general dependence is possible

\[ j(\varphi) = j_0 \sin(\varphi + \varphi_0). \]

**Rashba-type** spin-orbit coupling

\[ \alpha (\vec{\sigma} \times \vec{p}) \cdot \vec{n} \]

\( \vec{n} \) is the unit vector along the asymmetric potential gradient.
Geometry of the junction with BIS magnetic metal
\[ F = a |\Psi|^2 + \gamma |\vec{D}\Psi|^2 + \frac{b}{2} |\Psi|^4 - \varepsilon n \left[ \hbar \times \left( \Psi(\vec{D}\Psi)^* + \Psi^*(\vec{D}\Psi) \right) \right] \]

\[ D_i = -i \partial_i - 2eA_i \]

\[ a\Psi - \gamma \frac{\partial^2 \Psi}{\partial x^2} + 2i\varepsilon \hbar \frac{\partial \Psi}{\partial x} = 0, \]

\[ \Psi \propto \exp(i\tilde{\varepsilon}x) \exp(-x\sqrt{\frac{a-a_c}{\gamma}}), \quad \text{where } \tilde{\varepsilon} = \frac{\varepsilon \hbar}{\gamma} \]

\( \varphi_0 \) - Josephson junction (A. Buzdin, PRL, 2008).
\[ \Psi \propto \exp(i\tilde{\varepsilon}x)\exp(-x\sqrt{\frac{a-a_c}{\gamma}}), \quad \text{where} \quad \tilde{\varepsilon} = \frac{\varepsilon h}{\gamma} \]

In contrast with a \( \Pi \) junction it is not possible to choose a real \( \Psi \) function!
The phase shift $\phi_0$ is proportional to the length and the strength of the BIS magnetic interaction.

The $\phi_0$ Junction is a natural phase shifter.
Spontaneous flux (current) in the superconducting ring with $\Phi_0$ - junction.

$$E(\phi) = \frac{j_c}{2e} \left(-\cos(\phi + \phi_0) + \frac{k\phi^2}{2}\right)$$

$$k = \frac{c\Phi_0}{2\pi L j_c}$$

In the $k<<1$ limit the junction generates the flux $\Phi = \Phi_0(\phi_0/2\pi)$

$$\phi_o = \frac{2\varepsilon_h L}{\gamma}$$

*Very important*: The $\Phi_0$ junction provides a mechanism of a direct coupling between supercurrent (superconducting phase) and magnetic moment (z component).
Let us consider the following geometry:

\[ \varphi_0 = x \frac{v_{so}}{v_F} \frac{M_y}{M_0} \]

\[ \sin \theta = \frac{I}{I_c} \Gamma \quad \text{with} \quad \Gamma = \frac{E_J}{K \gamma} x \frac{v_{so}}{v_F} \]

voltage-biased Josephson junction

\[ \varphi(t) = \omega_J t \]
Magnetic anisotropy (easy z-axis) energy:

\[ E_M = -\frac{K\nu}{2} \left( \frac{M_z}{M_0} \right)^2 \]

Coupling parameter:

\[ \Gamma = \frac{E_J}{K\nu} x \frac{v_{so}}{v_F} \]

Weak coupling regime: \( \Gamma < 1 \).
Strong coupling regime: \( \Gamma > 1 \).

Let us consider first the \( \phi_0 \) - junction when a constant current \( I < I_c \) is applied:

\[ \varphi_0 = x \frac{v_{so}}{v_F} \frac{M_y}{M_0} \]

\[ E_{tot} = -\frac{\Phi_0}{2\pi} \varphi I + E_s (\varphi, \varphi_0) + E_M (\varphi_0) \]

Minimum energy condition:

\[ \partial_{\varphi} E_{tot} = \partial_{\varphi_0} E_{tot} = 0. \]
The current provokes rotation of the magnetic moment:

\[ M_y = M_0 \sin \theta \]

For the case \( \Gamma > 1 \) when \( |I| > \frac{I_c}{\Gamma} \) the moment will be oriented along the y-axis.

Applying to the \( \Phi_0 \) - junction a current (phase difference) we can generate the magnetic moment rotation.

a.c. current -> moment’s precession!
Magnetic moment precession – voltage-biased $\phi_0$- junction

$\varphi(t) = \omega_j t$

\[
\frac{dM}{dt} = \gamma M \wedge H_{\text{eff}} + \frac{\alpha}{M_0} \left( M \wedge \frac{dM}{dt} \right),
\]

where $H_{\text{eff}} = -\delta F/\mathcal{V} \delta M$ is the effective magnetic field

\[
H_{\text{eff}} = \frac{K}{M_0} \left[ \Gamma \sin \left( \omega_j t - r \frac{M_y}{M_0} \right) \hat{y} + \frac{M_z}{M_0} \hat{z} \right]
\]

$r = x v_{so}/v_F$
Landau-Lifshitz equation:

\[
\frac{dM}{dt} = \gamma M \wedge H_{\text{eff}} + \frac{\alpha}{M_0} \left( M \wedge \frac{dM}{dt} \right)
\]

\[H_{\text{eff}} = \frac{K}{M_0} \left[ \Gamma \sin \left( \omega_j t - r \frac{M_y}{M_0} \right) \hat{y} + \frac{M_z}{M_0} \hat{z} \right]\]

Magnetic moment precession:

\[
\frac{I}{I_c} = \sin \omega_j t + \frac{\Gamma r}{2} \frac{1}{\omega^2 - 1} \sin 2\omega_j t + \ldots,
\]
Complicated regime of the magnetic dynamics:

For more details – see (F. Konschelle and A. Buzdin, PRL, 2009).
Complementary Josephson logic

**RSFQ-logic using π-shifters**


**RSFQ-logic: Rapid Single Quantum logic**

Conventional RSFQ-cell

\[ LI_c > \Phi_0 \]

Fluxon memorizing cell

\[ L \rightarrow 0 \]

RSFQ- \( \pi \)-cell

\[ L_j = \Phi_0 / (2\pi I_c) \]

\[ \tau \sim 1 / (I_c R) \]

To operate at 20 GHz clock rate

\( I_c R \approx 50 \, \mu V \) has to be

We have \( I_c R > 0.1 \, \mu V \) for the present

\( \pi \)-RSFQ – Toggle Flip-Flop
Superconducting phase qubit

(a) Digital bit

(b) Quantum bit

|0⟩ | 1⟩

Φ_{ext} = Φ_0 / 2

or π-shift due to π-junction

π-junction

or π-shift due to π-junction

Φ_{ext} < 0.5 Φ_0

Φ_{ext} = 0.5 Φ_0

qubit operation
Conclusions

• Superconductor-ferromagnet heterostructures permit to study superconductivity under huge exchange field \( h>>T_c \).

• The \( \pi \) - junction realization in \( S/F/S \) structures is a quite general phenomenon.

• Domain wall superconductivity. Spin - valve effects.

• The BIS magnets provide a mechanism of the realization of the novel \( \varphi_0 \) - junctions with the very special properties.

• In these \( \varphi_0 \) - junctions a direct (linear) coupling between superconductivity and magnetism is realized. Seems to be ideal for superconducting spintronics.